Optimizing on-ramp entries to exploit the capacity of a road

M. Pica Ciamarra
Dip.to di Scienze Fisiche, Università di Napoli "Federico II," INFM-Coherentia, INFN and AMRA, Napoli, Italy
(Received 24 May 2005; revised manuscript received 26 August 2005; published 1 December 2005)

In this paper, we perform simulations of an extended Nagel-Schreckenberg model for one-lane and two-lane roads. We consider the presence of many entry ramps, placed in different locations, and we determine how the flux of cars of each entry ramp must be controlled in order to better exploit the capacity of the road. Our results are of relevance for the optimization of daily traffic, and set rules for the design of evacuation plans of urban conglomerations exposed to natural hazards, such as volcanoes.

DOI: 10.1103/PhysRevE.72.066102 PACS number(s): 89.40.−a, 45.70.Vn, 02.50.Ey

I. INTRODUCTION

Modern society relies on the efficient and correct functioning of communication and transportation over complex networks [1]. For instance, energy is transported over the power grid, information travels over the internet and over the telephone line, and people travel over the street networks of our nations.

Transportation problems have recently attracted the interest of physicist as they are related to the global behavior of systems with many elements interacting at short distances, such as bytes which travel over the internet, or cars traveling on the streets. In particular, highway vehicular traffic has been widely investigated both experimentally and theoretically (see [2] and references therein), and there is now evidence suggesting the existence of three qualitatively distinct dynamic states: the free traffic flow, the traffic jam, and the synchronized traffic flow [3].

Among the different methods of investigation and simulation of highway traffic cellular automata (CA) appear as the most promising. Due to their simplicity it is nowadays possible to simulate very large networks faster than realtime [4], keeping track of the movement of every single vehicle. However, it is only recently that a CA model able to reproduce the three qualitatively distinct dynamic states of highway traffic has been introduced [5–7]. This is an extension of the well known Nagel-Schreckenberg model [8], modified for the introduction of brake lights, anticipation and a slow to start rule.

In this paper by using this extended Nagel-Schreckenberg model (ENSM) we investigate the role of on-ramps. Previous works include simulations study of the interaction between on-ramp and main road with a simplified CA model [9] and with a discrete optimal velocity model [10], and analytical studies conducted in the framework of continuum traffic equations [11–13]. In particular, by studying an equation for vehicle density (with a sink/source term used to model the on-ramp) Helbing and co-workers [13] have shown that the upstream flux explores several dynamics states: free traffic flow, triggered stop and go waves, oscillatory congested traffic, homogeneous congested traffic, moving localized clusters and pinned localized clusters.

Here we extend these works in two aspects. First, we make simulations of both a one lane and a two lane road with the model which at the present better reproduces the experimental findings. Second, we consider the presence of more than a single on-ramp. Moreover, we study the problem of the interaction between an on-ramp and the main road with a new pragmatical perspective. The questions we want to answer are the following. Let us suppose we have an highway running near a large urban conglomeration, and that there are several accesses from the urban conglomeration to the highway. We want to understand how the accesses must be regulated in order to exploit the full capacity of the highway, i.e., in order to maximize the flux of cars running on the highway. For instance, is it better to close all the accesses but one, or to open two or three accesses? And if an access is open, how must the flux of entering cars be regulated?

These problems are of wide general interest, as it is notorious that highways near large urban conglomerations get easily congested during peak hours. They are also of primary importance in the ideation of evacuation plans of cities. For instance there are about half a million people living near the Plinian volcano Mt. Vesuvius, Italy, who need to be evacuated in case of an eruption forecast. With any probability these people will try to escape using the highway which runs in the area, and it is therefore necessary to devise strategies to maximize its flux.

The paper is structured as follows. We start with a short review of the ENSM, and of the rules used to model lane changing and on-ramps. Then, we consider the problem of flux optimization of a one lane road with one, two or three on-ramps. In this context the interaction between different on-ramps is investigated. Afterwards, the problem of flux optimization of a two lane road with many on ramps is also considered. Finally, some conclusions are drawn and possible improvements are discussed.

II. MODEL

A. The extended Nagel and Schreckenberg model

The ENSM is a cellular automata model for traffic flow. Here we shortly review the model, which has been studied in detail in Ref. [5]. With respect to the model used there here we introduce a new rule (rule 3b) which avoids unphysical decelerations (bigger than the gravitational acceleration); we will discuss the role of this rule later on. The model is characterized by six parameters. There are three braking probabilities which model the cases in which the preceding car is braking (p_b), the car is not moving (p_0), and all of the remaining conditions (p_d). The remaining three parameters are the range of interaction with the brake light of the preceding car h, a security distance d_s, and the maximum velocity $v_{\max}$. The road is dived in cell of length $L=1.5$ m, and each car occupies $l_c=5$ consecutive cells. Cars move from left to the right with a discrete velocity $v=0,1,\ldots,v_{\max}$. We use an updating time step of 1 s and a maximum velocity $v_{\max}=20=108$ km/h.

The state of a car n at time t is specified by its position x_n, its velocity $v_n(t)$, and the state of its braking lights $b_n(t)$ which can be activated $[b_n(t)=1]$ or deactivated $[b_n(t)=0]$. In order to describe the rules according to which the position of each car is updated, we introduce the following notations: d_n indicates the distance of car n from the preceding car, $d_n=x_{n+1}-x_n-l_c$; $t_h=d_n/v_n(t)$ and $t_s=\min\{v_n(t),h\}$ are the times needed to reach the position of the leading vehicle, and a velocity dependent temporal interaction horizon, respectively; $d_n^{(\mathrm{eff})}=d_n+\max\{v_{\mathrm{anti}}-d_s,0\}$ denotes the effective gap from the leading vehicle, where $v_{\mathrm{anti}}=\min\{d_{n+1},v_{n+1}\}$ is the expected velocity of the leading vehicle in the next time step. At every instant of time each car has a braking probability p_{dec} defined as follows:

$$p_{\mathrm{dec}}=p_{\mathrm{dec}}[v_n(t),b_{n+1}(t),t_h,t_s]=\begin{cases} p_b & \text{if } b_{n+1}=1 \text{ and } t_h<t_s,\\ p_0 & \text{if } v_n=0,\\ p_d & \text{in all other cases}.\end{cases} \tag{1}$$

The positions of the cars are updated in parallel according to the following rules ($t<t_1<t_2<t+1$).

(1) Determination of the braking probability:

$$p_{\mathrm{dec}}=p_{\mathrm{dec}}[v_n(t),b_{n+1}(t),t_h,t_s].$$

(2) Acceleration:
If $[b_{n+1}(t)=0$ and $b_n(t)=0]$ or $[t_h\geqslant t_s]$ then

$$v_n(t_1)=\min\{v_n(t)+1,v_{\max}\}.$$

(3) Deceleration:

$$v_n(t_2)=\min\{d_n^{(\mathrm{eff})},v_n(t_1)\}.$$

If $[v_n(t_2)<v_n(t)]$ then $b_n(t+1)=1$.

(3b) Deceleration control:

$$v_n(t_2)=\max\{v_n(t_2),v_n(t_1)-6\}.$$

(4) Randomization:
If $R<p_{\mathrm{dec}}$, $v_n(t+1)=\max\{v_n(t_2)-1,0\}$
else $v_n(t+1)=v_n(t_2)$.
If $p_{\mathrm{dec}}=p_b$ and $v_n(t+1)=v_n(t_2)-1$
then $b_n(t+1)=1$.

(5) Motion:

$$x_n(t+1)=x_n(t)+v_n(t+1).$$

Here R is a number picked form a uniform distribution in the range [0,1]. We use the following values for the parameter of the model, which have been proposed and validated in Ref. [5]: $p_b=0.94$, $p_0=0.5$, $p_d=0.1$, $h=6$, $d_s=7$, $v_{\max}=20$.

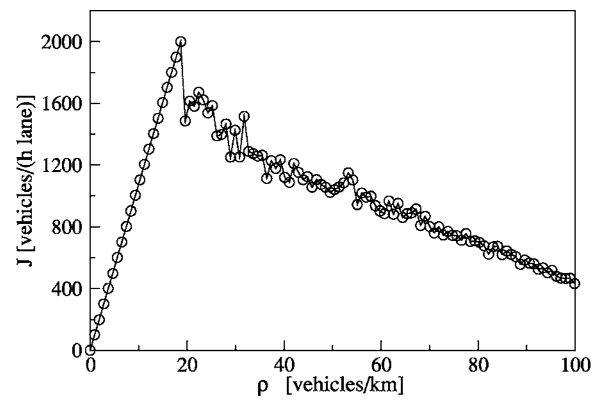

FIG. 1. Fundamental diagram of the ENSM described in Sec. II A. For $\rho<20$ vehic./km the traffic is free, and $J=\rho v_{\max}$. At higher density traffic is congested, and an increase of the density leads to a decrease of the total flux.

Rule (3b) assures that decelerations are never bigger than six cells/s^2=9 m/s^2, an estimate of maximum physically possible deceleration. The rule plays no role in the standard ENSM model with one or two lanes, as the system never moves to a state where accelerations bigger than six cells/s^2 occurs. It is useful when one consider on-ramps.

B. Periodic one lane road

Figure 1 shows the time-averaged fundamental diagram of the ENSM, that is the relation between the flux J of cars measured in a given position and the density of cars ρ, obtained in a simulation of a road of length $L=10000$ cells $=15$ km with periodic boundary conditions. When the density of cars is small each car is free to move at the maximum allowed velocity $v_{\max}$, and $J(\rho)=\rho v_{\max}$. As ρ increases cars start interacting until the breakdown density ($\rho\simeq 20$ vehic./km) is reached. At higher density the interaction between the cars is such that an increase in the density leads to a decrease of the total flux J. This is the congested flow regime, which is primarily controlled by the braking noise p_{dec}. The maximum flux $J\simeq 2000$ vehic./h is obtained with a density $\rho\simeq 20$ vehic./km.

C. Two lanes highway

In order to model two lane roads one has to introduce rules for lane changing [4], which are applied just before updating the positions of the cars. We use asymmetric rules (left→right rules and right→left rules are different) because in most countries faster cars travel on the left lane, and cars on the right lane cannot overtake cars on the left lane. In order to describe the applied rules, we consider a car n on a given lane, and we indicate with r (s) the car which follows (precedes) car n on the adjacent lane.

1. Overtake

A car n on the right lane moves to the left lane (overtake) if

(1) $[b_n(t)=0]$ and $[v_n(t)>d_n]$

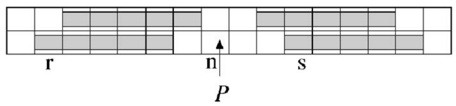

FIG. 2. The access to a road takes place into a single cell. We first check if it is possible to introduce a car without causing collisions. If this is the case the car is introduced with probability P.

(2) $[d_{ns}^{(\text{eff})} \geqslant v_n(t)]$ and $[d_{rn}^{(\text{eff})} \geqslant v_r(t)]$ and $d_{ns} > (v_n^2 - v_s^2)/12$,

where $d_{ij}^{(\text{eff})}$ indicates the effective distance between car i and car j. The first condition assures that a car attempts to move to the left lane only if it is not braking, and if it is obstructed by the leading car. The second condition is a safety criterion. Particularly, the condition $d_{ns} > (v_n^2 - v_s^2)/12$ assures that the distance between two consecutive cars is alway bigger than the security distance $(v_n^2 - v_s^2)/12$.

2. Return to right

A car n on the left lane returns to the right one if

(1) $(b_n(t)=0)$,

(2) $t_h^{ns} > 3$,

(3) $d_{rn} > v_r(t)$,

(4) $d_{ns} > (v_n^2 - v_s^2)/12$,

(5) $t_h > 6$ or $v_n > d_n$,

where $t_h^{ij} = d_{ij}/v_i(t)$. According to these rules a vehicle returns to the right lane if there is no disadvantage in regard to its velocity and it does not hinder any other vehicle by doing so [4].

D. On-ramp model

We model the access into the highway as happening in a single cell, as shown in Fig. 2. We first choice the velocity v_n of the entering car with respect to the distance from the leading car and to its velocity:

$$v_n = \min\{v_{\max}^{\text{imm}}, d_{ns}^{(\text{eff})}\}, \tag{2}$$

where $v_{\max}^{\text{imm}} = 11$ cells/s $\simeq 60$ km/h. Then, we check if it is possible to introduce a car with this velocity without causing any collision. This is done using the same rules adopted to verify if a car n can move to the left lane: $[d_{ns}^{(\text{eff})} \geqslant v_n(t)]$ and $[d_{rn}^{(\text{eff})} \geqslant v_r(t)]$ and $[d_r > (v_r^2 - v_n^2)/12]$. Finally, if the introduction of the car turns out to be safe, the car is introduced with probability P.

In this model of an on-ramp cars traveling on the highway have precedence over cars which want to enter the highway. $v_{\max}^{\text{imm}}$ plays the role of maximum allowed value for the velocity of an entering car, and models the fact that on-ramp cars are slower than highway cars. Via Eq. (2) we take into account the fact that cars entering the highway adapt their velocity to that of their leading vehicles. A similar adaptation scheme, which allows not to model explicitely on-ramp traffic as in [9], has been used in Ref. [10].

Since cars accelerate while on the on-ramp, increasing the maximum allowed entering velocity $v_{\max}^{\text{imm}}$ is qualitatively analogous to consider longer on-ramps. In the next section the role of the parameter $v_{\max}^{\text{imm}}$ is quantitatively investigated for a single lane road with a single on-ramp.

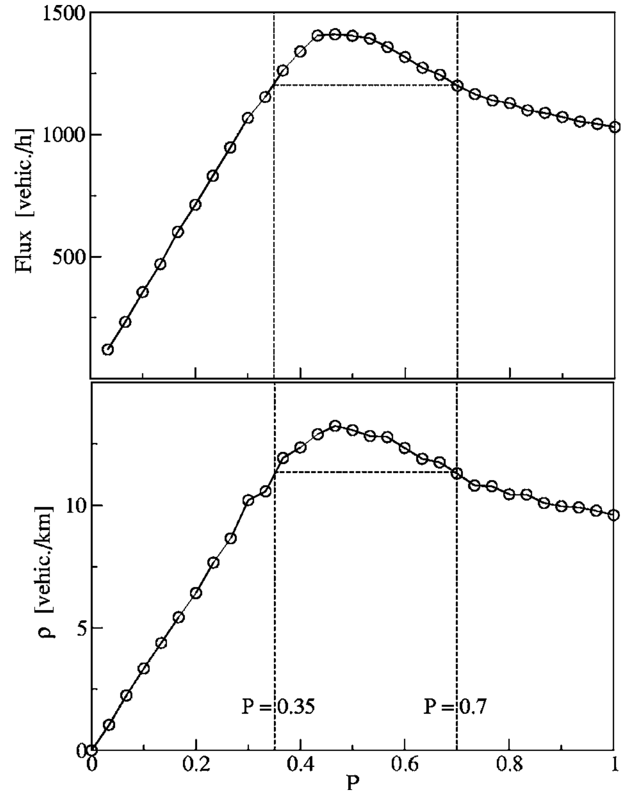

FIG. 3. Dependence of the flux (upper panel) and of the density (lower panel) of a single lane road with one entry ramp on the access probability P. The dashed lines put in evidence that the stationary states reached by the system with $P=0.35$ and $P=0.70$ are characterized by the same flux and the same density.

The parameter P plays an important role. Due to the interaction between entering cars and upstream flux, it is not possible to fix a priori the flux of entering cars. We exert via the parameter P the maximum allowed degree of control over this flux: when $P=0$ no cars enter, while for $P=1$ a car enters as soon as possible. We will show in the next section how the actual flux of entering car depends on P and on the value of the upstream flux.

III. ONE LANE ROAD

A. One on-ramp

We consider in the following the optimization of the flux in a one lane road. We start with the simplest case, that is a road of length $L=15$ km, with a single access placed at $x=0$: the upstream flux is $J=0$ as there are no cars for $x<0$. We use open boundary conditions, i.e., cars which reach the end of the road vanish. Figure 3 shows how the flux J (upper panel) and the density ρ (lower panel) depend on the access probability P. The maximum of J ($J \simeq 1400$ vehic./h), obtained for $P \simeq 0.5$, is much lower than the maximum flux allowed in a single lane road according to our model ($J \simeq 2000$ vehic./h, see Fig. 1). The density at which the maximum is attained is $\rho \simeq 14$ vehic./km, which is lower than the

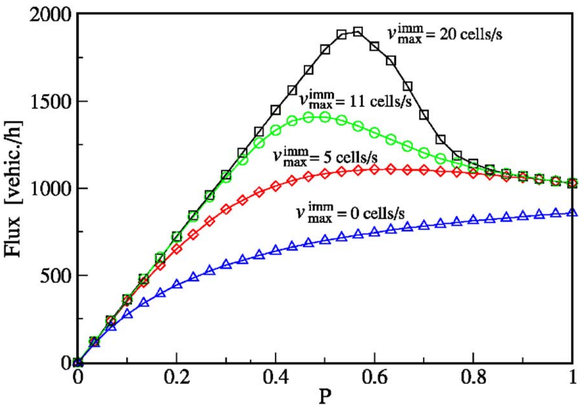

FIG. 4. (Color online) Dependence of the flux of a single lane road with one entry ramp on the access probability P, for $v_{max}^{imm}=0,5,11,20$. Higher values of v_{max}^{imm} models longer on-ramps. As v_{max}^{imm} increases $J(P)$ becomes not monotonic. Then, no qualitative changes appears.

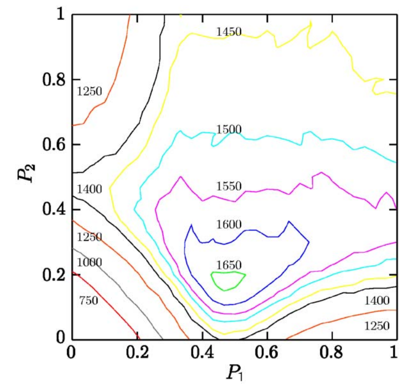

FIG. 5. (Color online) Contour level plot of the flux of a single lane road with two accesses, placed in $x=0$ km and in $x=7.5$ km. The two accesses are characterized by the parameters P_1 and P_2, respectively. The maximum flux is obtained for $(P_1,P_2) \simeq (0.45,0.18)$. For these values of the parameters the fluxes J_1 and J_2 at the two accesses are $J_1 \simeq 1380$ vehic./h and $J_2 \simeq 270$ vehic./h, respectively.

density $\rho \simeq 20$ vehic./km at which the flux is maximum. The apparent linear relation between density and flux is due to the fact that the density is always smaller than 20 vehic./km: a one lane road with a single entry is always in the free traffic flow region of the fundamental diagram of Fig. 1.

The nonmonotonic relation between the density and P is explained as follows. The number of cars (i.e., the density) which enter in a given time laps is given by $PN(P)$ where $N(P)$ is the number of time it happens that a vehicle can enter the road without causing any accident. $N(P)$ depends not only on the density but also on the spatial distribution of the vehicles: evidently the distribution attained for larger P is less favorable for the entry of new cars.

These numerical results show that in a one lane road the maximum flux one can achieve with a single on-ramp is 1400 vehic./km, with is attained when $P=0.5$. With no control on the on-ramp access, i.e., by allowing a car to enter the road as soon as possible ($P=1$), one obtains a smaller value of the flux. Since this maximum flux is much smaller than the maximum flux allowed in a single lane road, in the following we examine the possibility of increasing the flux by using more on-ramps.

The above results are expected to quantitatively depend on the on-ramp length, which in the present model is controlled by the maximum allowed value for the velocity of an entering car v_{max}^{imm}. In Fig. 4 we show the dependence of the flux on the access probability P for $v_{max}^{imm}=0$, 5, 11, and 20. For smaller values of v_{max}^{imm} the flux J monotonically increases with P. At higher values v_{max}^{imm} $J(P)$ becomes not monotonic, but no further qualitative changes are observed.

B. Two on-ramps

We discuss now the dependence of the flux J measured at the end of a single lane road of length $L=15$ km with open boundary conditions, on the access probabilities P_1 and P_2 of two on-ramps placed respectively at $x=0$ and $x=7.5$ km. The upstream flux (i.e., the flux for $x<0$) is zero. A contour level plot of $J(P_1,P_2)$ is shown in Fig. 5. Due to the asymmetry of the rules regulating the entry of new cars, which give precedence to car travelling on the street, the two on-ramps are not equivalent, i.e., $J(P_1,P_2) \neq J(P_2,P_1)$. The equivalence between the two on-ramps is recovered in the particular case in which one access is closed ($P=0$), i.e., $J(P,0)=J(0,P)$. For a fixed value of P_1 (P_2) by increasing P_2 (P_1) first the total flux grows until it reaches a maximum at a particular value of $P_2^{max}(P_1)$ $[P_1^{max}(P_2)]$, and then it decreases reaching the minimum at $P_2=1$ ($P_1=1$), as shown in Fig. 6. The maximum flux $J \simeq 1650$ vehic./h is obtained for $(P_1,P_2) \simeq (0.45,0.18)$, when the fluxes of cars entering the road are $J_1 \simeq 1380$ vehic./h and $J_2 \simeq 270$ vehic./h, respectively. While one may expect that when the total flux is maximum the flux at the first entrance is bigger than the flux at the second one, it is surprising the large disparity between the two fluxes, which differ by a factor of 5.

The interaction between the two ramps can be understood from the results of Fig. 7. Here we show the trajectories of the vehicles in the range 5.5 km$<x<$9.5 km, which includes the position of the second access ($x=7.5$ km). In the

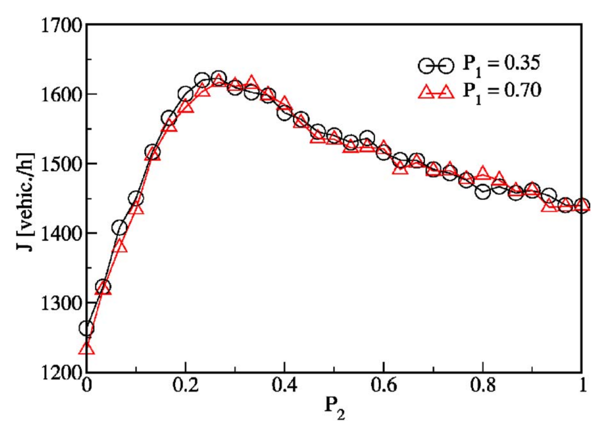

FIG. 6. (Color online) Dependence of the flux on the value of the access probability P_2 of the second ramp, for fixed values of the access probability P_1 of the first ramp.

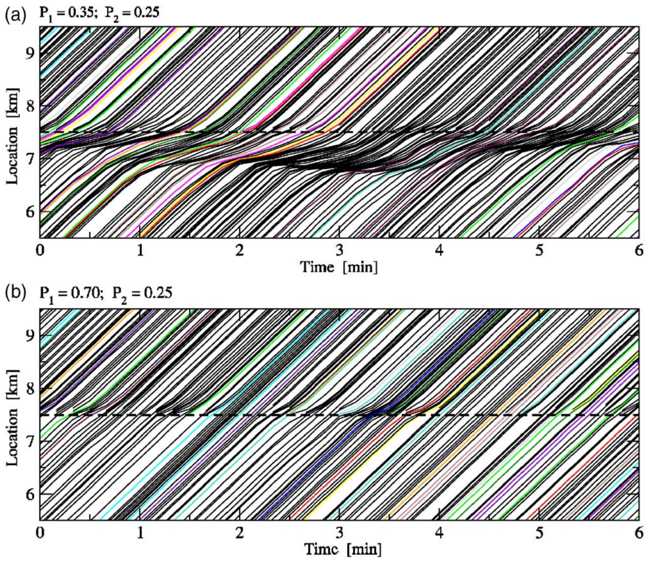

FIG. 7. (Color online) Trajectories of cars moving on a single lane road with two accesses, the second one located at $x=7.5$ km (dashed lines). In the case $P_1=0.35$ (upper panel) a stationary jam forms just before the second entrance, while when $P_1=0.70$ (lower panel) no jam appears. The fluxes of cars entering at the first entrance in the two cases are equal, as well as the fluxes of cars entering at the second one.

upper panel we consider the case $(P_1,P_2)=(0.35,0.25)$, in the lower one the case $(P_1,P_2)=(0.70,0.25)$. We observe that in the first case the trajectories for $x<7.5$ km are strongly influenced by the presence of the on-ramp, as they start bending (i.e., vehicles slow down) about half kilometer before the on-ramp. There is a stationary jam in the range $x=7-7.5$ km. In the second case, when $P_2=0.70$, no jam appears: cars at the second access succeed in merging with the upstream flux without causing major disturbances.

Note that in the two cases we are comparing, $(P_1,P_2)=(0.35,0.25)$ and $(P_1,P_2)=(0.70,0.25)$, both the upstream fluxes and the downstream fluxes are equals. The differences in the trajectories of the vehicles depend on the spatial properties of the upstream fluxes, which indeed are different in the two considered cases, as evidenced by the time headway distribution (not shown).

The interaction between the ramps depend on the upstream flux and on the entering flux. For a given value of P_1, as P_2 increases we observe a transition from a state in which the fluxes only slightly interact, and the trajectories look like those shown in Fig. 7(b), to a state in which the fluxes interact and the trajectories look like those shown Fig. 7(a).

C. Three on-ramps

We consider now the problem of maximising the flux of a one lane road with three on-ramps. To this end we have simulated a road of length $L=1200$ cells$=18$ km, with three on-ramps placed at $x=0$, 6 and 12 km. The flux J is now a function of the access probabilities P_1, P_2, and P_3. Figure 8 shows the dependence of the flux on (P_2,P_3) for $P_1=0.17$, 0.33, 0.50, 0.67, 0.83, and 1. When P_1 is small the second ramp is only slightly influenced by the presence of the first one, $J(P_2,P_3)$ is very similar to that of a single lane road with two ramps, and the maximum flux is obtained again for $(P_2,P_3)\simeq(0.45,0.18)$. As P_1 grows more cars enter the road at the first ramp, influencing the dependence of the total flux on P_2 and P_3. For instance the maximum flux $J\simeq 1745$ vehic./km is obtained for $(P_1,P_2,P_3)=(0.67,0.3,0.1)$, when the fluxes of cars entering at each access are $(J_1,J_2,J_3)=(1239,357,149)$ vehic./h.

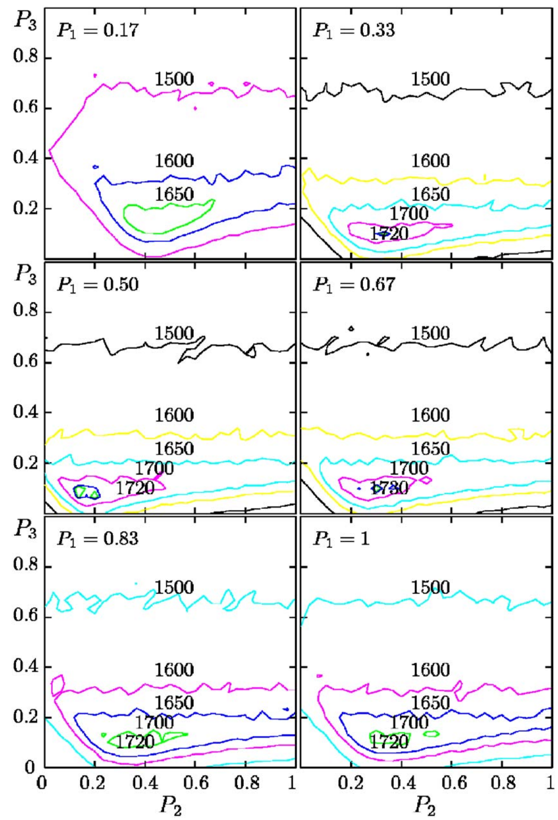

FIG. 8. (Color online) Contour plot of the flux J of a single lane road with three accesses with access probabilities P_1, P_2, and P_3, respectively. Each plot shows the dependence of J on P_2 and P_3 for a fixed value of P_1.

D. Discussions

We would like to put in evidence the particular form of interaction between the accesses. Contrary to a naive expectation our results show that, given a road with more than one access, in order to optimize the total flux one cannot simply optimize the fluxes of cars entering at each on-ramp, starting with the first one. This result is apparent from Fig. 9. For instance, the optimal value of the flux at the first entry J_1 varies (decreases) when the number of entries increases. On the contrary, J_2 increases.

While we have not conducted studies with more than three entries, we believe that the addition of more accesses can only slightly increase the total flux. For instance the difference $\Delta J_{(i,j)}$ between the fluxes obtained with i and with j accesses are $\Delta J_{(1,2)}=1650-1400=250$ vehic./h, and

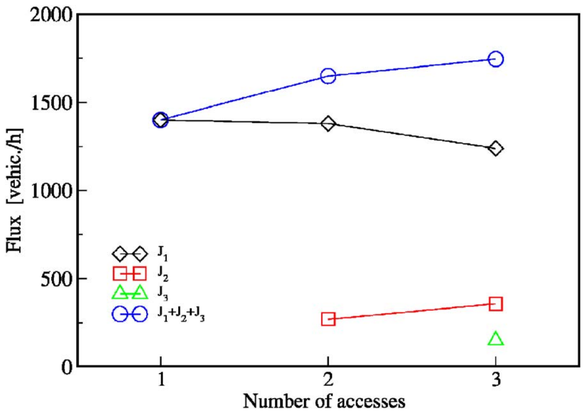

FIG. 9. (Color online) Dependence of the optimal flux of cars entering at each access in a single lane road as a function of the number of accesses. For a given number of accesses, $J_{max}=J_1+J_2+J_3$ is the maximum flux of cars one can achieve.

$\Delta J_{(2,3)}=1720-1650=70$ vehic./h. Of course, the more accesses are included, the smaller the increase of the total flux that each access will cause. The particular values of $\Delta J_{(1,2)}$ and of $\Delta J_{(2,3)}$ suggest that the maximum flux achieved with many accesses, $J_{max}=\Sigma_{i=0}^{\infty}\Delta J_{(i,i+1)}$, will be smaller that the maximum value of the flux the model allows in a single lane road.

IV. TWO LANE ROAD

We discuss now the optimization of the flux of a two lane road. The rules for lane changing are given in Sec. II C.

A. One on-ramp

Figure 10 shows the dependence of the flux of a two lane road with one on-ramp on the access probability P. For comparison, the result obtained in the case of a single lane road is also shown. The shapes of the two curves are very similar, but the maximum flux reached in a two lane road ($J \simeq 1650$) vehic./h is bigger than that reached in a single lane road ($J \simeq 1400$ vehic./h). This result suggests that maximum

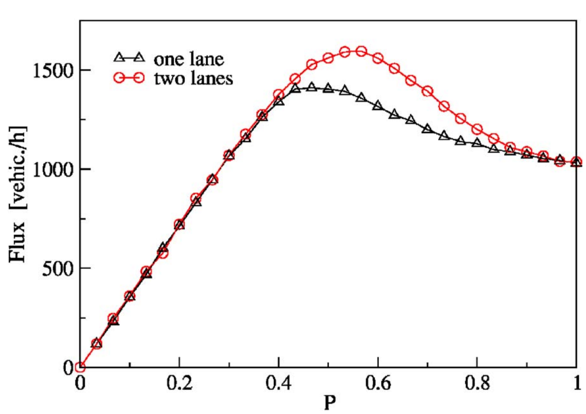

FIG. 10. (Color online) Dependence of the flux of a two lane road (circles) and of a single lane road (triangles) with one access on the access probability P.

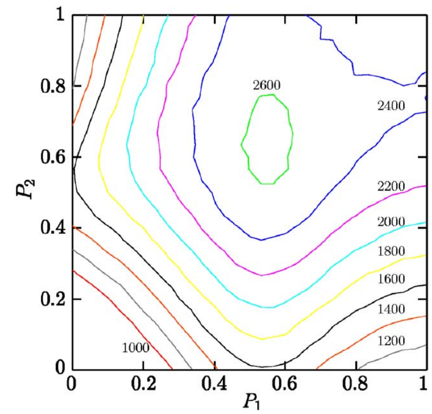

FIG. 11. (Color online) Contour level plot of the flux of a two lane road with two accesses, placed in $x=0$ km and in $x=7.5$ km. The two accesses are characterized by the parameters P_1 and P_2, respectively. The maximum flux is obtained for $(P_1,P_2) \simeq (0.60,0.65)$. For these values of the parameters the fluxes J_1 and J_2 at the two accesses are $J_1 \simeq 1565$ vehic./h and $J_2 \simeq 1047$ vehic./h, respectively.

flux obtained in a single lane road is limited by the ramp-road interaction, and not by the capacity of the on-ramp. As in the case of a single lane road if no control is exerted on the entering flux, i.e., if $P=1$ and a car enters as soon as possible, one does not obtain the optimal flux.

B. Two on-ramps

The dependence of the flux on the access probabilities in the case of a two lane road with two on-ramps is shown in Fig. 11. The qualitative behavior is similar to that observed in the case of a single lane road, and also in this case the two ramps are not equivalent, i.e., $J(P_1,P_2) \neq J(P_2,P_1)$. The maximum flux $J=2612$ vehic./h is achieved with $(P_1,P_2)=(0.60,0.65)$, when the flux of cars at the two entrances are $J_1=1565$ vehic./h and $J_2=1047$ vehic./h. With respect to the case of a road with a single lane, here the fluxes of entering cars differ by a small factor ($\simeq 1.5$). This is due to the fact that while with a single lane two entries are able to create a flux which is around 82% of the maximum one (1650 out of 2000 vehic./h), in a two lane road the percentage is much smaller, 65% (2612 out of 4000 vehic./h). With this small value of the flux car approaching the second entrance are most of the time free to move to the left lane if obstructed, as confirmed by Fig. 12. Here we plot the spatial variation of the fraction of cars which occupy the right (n_r) and the left ($n_l=1-n_r$) lane, near the position of the second entrance. Before the entrance the density of cars is $\rho \simeq 7.3$ vehic./(km lane) and n_r is bigger than n_l, while after the entrance the density is $\rho \simeq 12.2$ vehic./(km lane), and n_r becomes smaller than n_l. This result confirms the well known "lane inversion" effect, according to which at higher densities most of the cars occupy the left lane, even though driving rules "push" cars on the right one.

C. Three on-ramps

Let us consider now the case of a two lane road with three on ramps, placed at $x=0$, 6.75, and 13.5 km. Figure 13

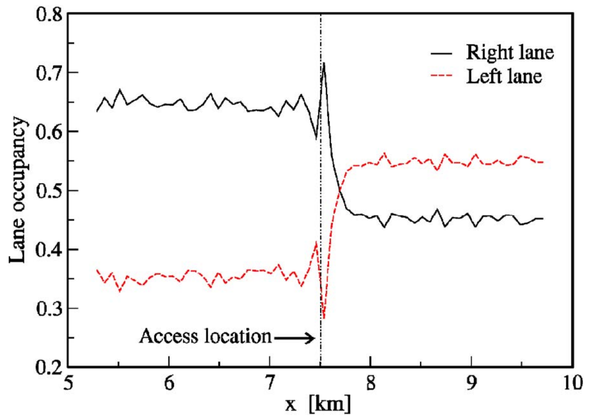

FIG. 12. (Color online) Space dependence of the fraction of cars which occupy the right and the left lane in a two lane road with two entries, characterized by $P_1=0.60$ and $P_2=0.65$, respectively. A sharp transition occurs from a state in which most of the cars are on the right lane to a state in which most of the cars are on the left lane. The transition is located near the position of the second entrance ($x=7.5$ km).

shows the dependence of the flux on P_2 and P_3 for $P_1=0$, 0.2, 0.4, 0.6, 0.8, and 1. Again, the result is qualitatively similar to that obtained in the case of a single lane road, but the values of the total flux are higher. The maximum flux 3245 vehic./h is obtained for $(P_1,P_2,P_3)=(0.6,0.7,0.43)$,

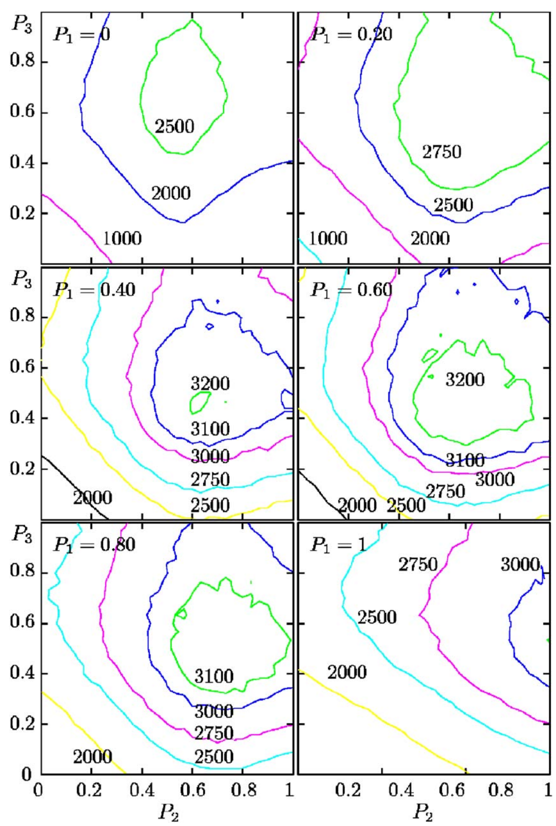

FIG. 13. (Color online) Contour plot of the flux J of a two lane road with three on-ramps with access probabilities P_1, P_2, and P_3, respectively. Each plot shows the dependence of J on P_2 and P_3 for a fixed value of P_1.

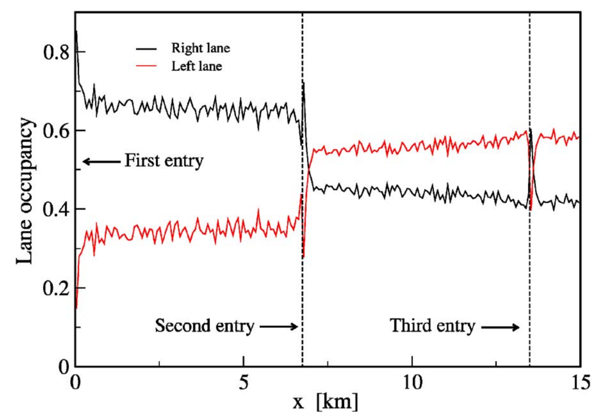

FIG. 14. (Color online) Space dependence of the fraction of cars which occupy the right and the left lane in a two lane road with three entries, characterized by $P_1=0.60$, $P_2=0.70$, and $P_3=0.45$, respectively. A sharp transition occurs from a state in which most of the cars are on right lane to a state in which most of the cars are on the left lane. The transition is located near the position of the second entrance ($x=6.75$ km). The third entry has no permanent effect on the vehicle distribution on the two lanes.

when the fluxes of entering cars are $(J_1,J_2,J_3)=(1550,1049,646)$ vehic./km. As in the case of two on-ramps, there is a sharp transition form a state in which most of the cars are on the right lane to a state in which most of the cars are on the left one, which is located near the position of the second entry. The fraction of cars on the two lanes is instead only slightly modified by the third entry, as shown in Fig. 14.

D. Discussion

As in a single lane road also in the case of a two lane road the problem of flux optimization in the presence of more than one entry turns out to be a collective problem. Figure 15 shows the optimal choice for the fluxes of entry cars at each on-ramp in the case of a two lane road with one, two, or three entries. It is evident that the optimal flux at the first (or

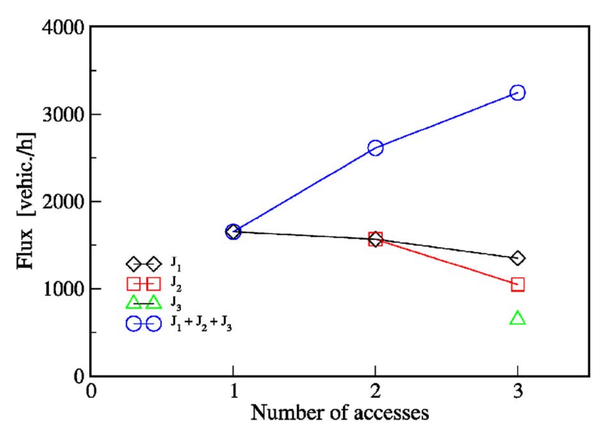

FIG. 15. (Color online) Dependence of the optimal flux of cars entering at each access in a two lane road as a function of the number of accesses. For a given number of accesses, $J_{max}=J_1+J_2+J_3$ is the maximum flux of cars one can achieve.

second) ramp depends on the total number of ramps. Figure 15 also shows the maximum flux one can achieve given the number of entry ramps. The maximum flux with three ramps is 3245 vehic./h, which is about 82% of the maximum flux allowed in a two lane road. We do expect that this maximum flux can be increased by using more entries.

V. HOW MUCH OPTIMIZATION

In order quantify the degree of optimization achieved via a control of the fluxes of car entering a road in different locations, one can compare the optimal flux J_{opt} obtained with particular vales of the entering probabilities P, with the flux J_{free} obtained if no control over the entering flux is exerted (i.e., $P=1$). The degree of optimization is quantified by $\delta=(J_{\mathrm{opt}}-J_{\mathrm{free}})/J_{\mathrm{free}}$. In a one lane road J_{opt} turns out to be 60% higher than J_{free} in the case of a single entry, around 15% higher in the case of two and three entries. In a two lane road J_{opt} is 35% higher than J_{free} in the case of a single entry, around 20% higher in the case of two and three entries. Via a control of the fluxes of cars entering a road in different locations it is therefore possible to obtain a remarkable improvement of the total flux.

VI. CONCLUSIONS

In this paper we have studied how it is possible to optimize the flux of cars running on a road via the control of the fluxes which enter the road at the various on-ramps. Precisely, we have studied this optimization problem in the case of one lane and of a two lane road, in the presence of one, two, or three entry ramps.

Our results suggest that, in the case of a single lane road, three carefully controlled on-ramps allows to exploit the road capacity almost at the best (86%). In this case we do not expect that a much higher value of the flux can be obtained by increasing the number of entries as (1) an entry ramp will always be a bottelneck for the upstream flux, and (2) entering cars have always a velocity which is smaller than that needed to obtain the maximum flux allowed by the road.

In the case of a two lane road with three on-ramps the road capacity is exploited at (82%): it is probably possible to increase this percentage via the use of more entry ramps.

The most important result of this work is the following: if a road has severals on-ramps R_1, $R_2, \ldots, R_n$ in locations $x_1 < x_2 < \cdots < x_n$, in order to maximize the total flux (i.e., the flux for $x > x_n$) one cannot optimize first the flux of cars entering at R_1, then that entering at R_2 and so on. Due to an effective interaction between the fluxes at the various on-ramps, one must optimize all of the fluxes at the same time.

Extension of this work includes a more accurate description of the on-ramp, both to take into account in a more precise way the road-ramp interaction, and to take into account the existence of ramps with acceleration lanes of different length.

ACKNOWLEDGMENTS

I thank Antonio Coniglio for rousing my interest in this problem and for helpful discussions. Work supported by EU Network MRTN-CT-2003-504712, MIUR-PRIN 2004, MIUR-FIRB 2001, CrdC-AMRA, INFM-PCI.

[1] L. Zhao, Y. C. Lai, K. Park, and N. Ye, Phys. Rev. E **71**, 026125 (2005).
[2] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001).
[3] B. S. Kerner and H. Rehborn, Phys. Rev. E **53**, R1297 (1996); **53**, R4275 (1996); Phys. Rev. Lett. **79**, 4030 (1997); B. S. Kerner, Phys. Rev. Lett. **81**, 3797 (1998); J. Phys. A **33**, L221 (2000).
[4] S. F. Hafstein, R. Chrobok, A. Pottmeier, M. Schreckenberg, and F. C. Mazur, Comput. Aided Civ. Infrastruct. Eng. **19**, 338 (2004).
[5] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **70**, 016115 (2004).
[6] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, J. Phys. A **33**, L477 (2000).
[7] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **65**, 015101(R) (2001).
[8] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).
[9] R. Jiang, Q. S. Wu, and B. H. Wang, Phys. Rev. E **66**, 036104 (2002).
[10] P. Berg and A. Woods, Phys. Rev. E **64**, 035602(R) (2001).
[11] H. Y. Lee, H.-W. Lee, and D. Kim, Phys. Rev. E **59**, 5101 (1999).
[12] B. S. Kerner and S. L. Klenov, Phys. Rev. E **68**, 036130 (2003).
[13] D. Helbing, A. Hennecke, and M. Treiber, Phys. Rev. Lett. **82**, 4360 (1999).